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Patterns in open string field theory solutions

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Abstract: In open string field theory the kinetic operator mixes matter and ghost sectors, and thus the ghost structure of classical solutions is not universal. Nevertheless, we have found from numerical analysis that certain ratios of expectation values for states involving pure ghost excitations appear to be universal. We give an analytic expression for these ratios and find good evidence that they are common to all known solutions of open string field theory, including the tachyon vacuum solution, lump solutions and string fields representing marginal deformations. We also draw attention to a close correspondence between the expectation values for the pure matter components in the tachyon vacuum solution and those in the solution of a simpler equation for a ghost number zero string field. Finally we observe that the action of L_0 on the tachyon condensate gives a state that is approximately factorized into a matter and a ghost part.

Keywords: Bosonic Strings, D-branes.

Contents

1. Introduction and summary

During the last few years many different kinds of solutions of open cubic string field theory [1] (OSFT) have been studied numerically. These include the tachyon vacuum solution [2], tachyon lump solutions describing various D-branes [3, 4], and solutions representing marginal deformations [5]. So far, however, there has been little progress in obtaining the analytic form of any of these solutions (see [6] for some attempts). This is the problem that motivates the work presented in this paper.

Analytic solutions are much easier to obtain in vacuum string field theory (VSFT), a version of open string field theory proposed to describe physics around the tachyon vacuum (see [7] and references cited therein, and [8, 9, 10, 11] for some recent developments). Nevertheless, there are several reasons that analytic solutions to OSFT are desirable. As presently formulated VSFT requires an extremely simple but singular kinetic term. An OSFT solution for the tachyon vacuum would most likely allow a derivation of VSFT, and may show if there is a simple regular form of the theory. In doing so the solution would allow a clear and complete proof of the tachyon conjectures [12]. Moreover, solutions of OSFT are likely to teach us interesting and useful facts about the open string star algebra and its interplay with the BRST and/or Virasoro operators. While solutions of VSFT have recently taught us a lot about star algebra projectors, more tools seem necessary to write OSFT solutions.

In OSFT the string field is represented by a ghost number one state in the state space of the combined matter-ghost conformal field theory. Since physics around different classical solutions (other than possibly the vacuum solution) are described by different matter conformal field theories with a common ghost system, one might have naively expected that these different classical solutions will have a common ghost structure, and will differ from each other in their matter part. While this is the case in VSFT, and is a direct consequence of its simplicity, it is not the case in OSFT as can be easily seen by examining the various numerical solutions obtained so far. One could hardly have expected this structure since the kinetic operator of OSFT mixes non-trivially matter and ghost sectors. Nevertheless one could ask if some part of the ghost structure is universal, namely, independent of which classical solution we are considering. This is one of the questions we shall address.

We show in section 2 that if the coefficient of any state in the string field theory solution has such universal structure, then it is easy to find the analytic expression for this coefficient by examining the solution describing small marginal deformations. Marginal deformations are associated with a dimension one boundary operator in the matter conformal field theory. To first order in the deformation parameter the solution representing such deformation is represented by only one state, obtained by the product of this dimension one matter primary and the ghost ground state of dimension minus one. To second order in the deformation parameter various other states are excited, but the coefficients of each of these states can easily be found by explicitly solving the string field equations to this order. This gives a way to compute the *ratios* of the coefficients of various states in this solution to this order. If any of these ratios is universal, then this computation determines this universal number.

While this tells us how to compute a universal ratio, it does not tell us which ratios are likely to be universal. For this we rely on numerical results. In section 3 we take various solutions (vacuum solution, lump solutions and solutions representing marginal deformations) and compare the coefficients appearing in these solutions to determine which of these coefficients are universal. Our analysis indicates that the ratios of the coefficients of the state $c_{-n}b_{-m}c_1|0\rangle$ to that of $c_1|0\rangle$ may be universal for any pair of *odd* integers m, n. The explicit analytical expression for this ratio can then be computed using the method outlined above and is given by $n\widetilde{N}^{11}_{mn}/(n+m-1)$, where $\widetilde{N}_{mn}$ are the ghost Neumann coefficients introduced in ref.[13]. We are thus led to believe that the string field $|\Psi\rangle$ representing a classical solution of OSFT contains a piece

$$|\Psi\rangle = \alpha_0 \left(1 + \sum_{n,m=\mathrm{odd}} \frac{n\widetilde{N}^{11}_{mn}}{(m+n-1)} c_{-n}b_{-m}\right) c_1|0\rangle + \cdots . \tag{1.1}$$

While an infinite series of ratios are fixed, the overall constant α_0 is solution dependent. An immediate consistency check is possible. It was shown in [14, 15] that a solution of the string field equations in the Siegel gauge must be singlets of an $\mathrm{SU}(1,1)$ symmetry acting on the ghost sector. Since, $\widetilde{N}^{11}_{mn} = \widetilde{N}^{11}_{nm}$ for the cubic vertex of OSFT, it is clear from the above expression that this sector of the string field is built from linear combinations of the form $(nc_{-n}b_{-m} + mc_{-m}b_{-n})$ and these are indeed $\mathrm{SU}(1,1)$ singlets [14, 15]. Of course, the prediction in (1.1) goes far beyond $\mathrm{SU}(1,1)$ in that it prescribes specific linear combinations of those singlets.

Given that the string field equations are non-linear, the equations of motion of the proposed universal coefficients will receive contribution from the non-universal coefficients. Thus the universality of the type proposed here may seem highly unnatural. However, it will be natural if these coefficients were determined by a set of linear equations satisfied by the string field. Although at present we do not know of any way to derive all the universal coefficients this way, we show in section 4 that there are some linear equations which any solution of the string field theory equations of motion is expected to satisfy. Some particular relations involving the universal coefficients follow from these linear equations.

Sections 5 and 6 are devoted to the study of some other aspects of the solutions of OSFT equations of motion. In vacuum string field theory solutions of classical equations of motion are factorized into a product of a matter part and a ghost part. This is not the case in OSFT since the kinetic operator Q_B involves both matter and ghost parts. Nevertheless we show in section 5 that the numerical results for the solution $|\Psi_0\rangle$ representing the tachyon vacuum has the property that $L_0|\Psi_0\rangle$ is approximately factorized into a product of a matter part and a ghost part.

In section 6 we draw attention to a surprising feature of the numerical results for the pure matter excitations in the tachyon condensate $|\Psi_0\rangle$ representing the vacuum solution. These excitations are given by the action of the matter Virasoro generators on the ghost number one ground state $c_1|0\rangle$. Denoting by $\mathcal{O}_m$ any particular combination of matter Virasoro generators, we find that the ratio of the coefficient of $\mathcal{O}_m c_1|0\rangle$ to that of $c_1|0\rangle$ in $|\Psi_0\rangle$ is very close to the ratio of $\mathcal{O}_m|0\rangle$ to $|0\rangle$ in the solution of another equation:

$$(L_0 - 1)|\Phi\rangle + |\Phi * \Phi\rangle = 0\,, \tag{1.2}$$

where $|\Phi\rangle$ is a ghost number zero string field. The correspondence is tested using numerical solutions to the field equations to level (10,30) in appendix A.

2. Computation of the universal coefficients

We consider formulating string field theory in a background represented by a matter boundary conformal field theory with an exactly marginal deformation. Associated with this deformation is a dimension one matter primary field V. Let $|V\rangle_m$ denote the corresponding state in the Hilbert space of the matter CFT. Then to first order in the deformation parameter λ, the solution of the OSFT field equations representing this deformation is given by:

$$|\Psi\rangle = \lambda c_1|0\rangle_g \otimes |V\rangle_m \equiv \lambda|\chi^{(1)}\rangle\,, \tag{2.1}$$

where $|0\rangle_g$ denote the SL(2,R) invariant ghost vacuum. $|\Psi\rangle$ given above satisfies the OSFT field equation:

$$Q_B|\Psi\rangle + |\Psi * \Psi\rangle = 0\,, \tag{2.2}$$

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to order λ. Let us now denote the solution to the OSFT field equation to second order in λ by

$$|\Psi\rangle = \lambda\chi^{(1)}\rangle + \lambda^2\chi^{(2)}\rangle\,. \tag{2.3}$$

Substituting this into eq. (2.2) and collecting terms to second order in λ we get,

$$Q_B|\chi^{(2)}\rangle = -|\chi^{(1)} * \chi^{(1)}\rangle\,. \tag{2.4}$$

If we take $|\Psi\rangle$ to be in the Siegel gauge,

$$b_0|\Psi\rangle = 0\,, \tag{2.5}$$

then, by applying b_0 on both sides of the equation we get:

$$L_0|\chi^{(2)}\rangle = -b_0|\chi^{(1)} * \chi^{(1)}\rangle\,. \tag{2.6}$$

We can now easily solve for $\chi^{(2)}$ by expressing it as an arbitrary linear combination of ghost number one states and comparing the two sides of the above equation. One particular consistency condition required for this equation to have a solution is that $b_0|\chi^{(1)} * \chi^{(1)}\rangle$ should not contain any state of vanishing L_0 eigenvalue. This in turn is related to the condition that V is an exactly marginal operator in the matter CFT.

The right hand side of eq. (2.6) can be evaluated as follows. We write:

$$b_0^{(3)}|\chi^{(1)} * \chi^{(1)}\rangle_{(3)} = (|V\rangle_m *_m |V\rangle_m)_{(3)} \otimes \,_{(1)}\langle 0|c_{-1}^{(1)}\,\,_{(2)}\langle 0|c_{-1}^{(2)}\; b_0^{(3)}|V_{123}\rangle\,, \tag{2.7}$$

where $|V_{123}\rangle$ is the ghost vertex, given by [13]:

$$|V_{123}\rangle = \exp\left(\sum_{r,s=1}^{3}\sum_{\substack{m\geq 0\\ n\geq 1}} \widetilde{N}^{rs}_{mn} n b^{(r)}_{-m} c^{(s)}_{-n}\right) c_0^{(1)}c_1^{(1)}|0\rangle_{(1)} \otimes c_0^{(2)}c_1^{(2)}|0\rangle_{(2)} \otimes c_0^{(3)}c_1^{(3)}|0\rangle_{(3)}\,. \tag{2.8}$$

Here the $\widetilde{N}^{rs}_{mn}$ are Neumann coefficients. They have cyclic symmetry $(r,s) \to (r+1, s+1)$ with $r+3 \equiv r$, $s+3 \equiv s$. In the matter sector the star product gives

$$|V\rangle_m *_m |V\rangle_m = \alpha|0\rangle_m + \cdots\,, \tag{2.9}$$

where α is some constant, and $\cdots$ denote excited states in the matter sector. The effect of the ghost vacua $\langle 0|c_{-1}$ in the right hand side of (2.7) is to delete from the ghost part of the vertex all reference to oscillators in the first and second state spaces. Thus

$$b_0^{(3)}|\chi^{(1)} * \chi^{(1)}\rangle_{(3)} = (\alpha|0\rangle_m + \cdots)_{(3)} \otimes \exp\left(\sum_{m,n\geq 1} \widetilde{N}^{11}_{mn} n b^{(3)}_{-m} c^{(3)}_{-n}\right) c_1^{(3)}|0\rangle_{(3)}\,. \tag{2.10}$$

Using eqs. (2.6), (2.10), and the equation:

$$L_0 c_{-n} b_{-m} c_1|0\rangle = (m+n-1) c_{-n} b_{-m} c_1|0\rangle\,, \tag{2.11}$$

we find the coefficient $\alpha_{n,m}$ of the state $c_{-n}b_{-m}c_1|0\rangle$ in $|\chi^{(2)}\rangle$ to be:

$$\alpha_{n,m} = \frac{n}{m+n-1}\,\widetilde{N}^{11}_{mn}\,\alpha\,. \tag{2.12}$$

On the other hand, the coefficient of the $c_1|0\rangle$ term, obtained by keeping the zeroth order term in the expansion of the exponential in eq. (2.8), is

$$\alpha_0 = \alpha\,. \tag{2.13}$$

Thus we get the ratio of these terms to be

$$r_{n,m} \equiv \frac{\alpha_{n,m}}{\alpha_0} = \frac{n}{m+n-1}\,\widetilde{N}^{11}_{mn} \equiv \bar{r}_{n,m}\,. \tag{2.14}$$

Although the ratio $r_{n,m}$ has been computed for a solution of OSFT representing a marginal deformation with small deformation parameter, if this ratio is universal then $\bar{r}_{n,m}$ as defined in (2.14) will represent the ratio of the coefficients of $c_{-n}b_{-m}c_1|0\rangle$ and $c_1|0\rangle$ in any solution of the string field theory equations of motion. In the next section we shall examine the numerical results and show that these ratios do appear to be universal provided m and n are *odd* integers.

3. Test of universality of the coefficient of $c_{-n}b_{-m}c_1|0\rangle$

In this section we present numerical evidence that the ratio $r_{m,n}$ of the coefficients of $c_{-n}b_{-m}c_1|0\rangle$ and $c_1|0\rangle$ is universal, independent of which solution we analyze. We do this by evaluating these coefficients for various solutions obtained by using level truncation, and showing that in each case the result comes close to the prediction (2.14).

For the explicit prediction we need the Neumann coefficients $\widetilde{N}^{11}_{mn}$. We have:

$$\begin{aligned}
\widetilde{N}^{11}_{mn} &= \frac{2}{3}\frac{(-1)^{m+1}}{n^2-m^2}\,(nA_mB_n - mA_nB_m)\,, && m+n = \text{even}\,,\ m\neq n\,,\\
\widetilde{N}^{11}_{mn} &= 0\,, && m+n=\text{odd}\,,\\
\widetilde{N}^{11}_{nn} &= \frac{1}{3n}(-1)^n\left(2S(n) - 1 - (-1)^n A_n^2 - 2A_nB_n\right), && S(n) = \sum_{k=0}^{n}(-1)^k A_k^2\,.
\end{aligned} \tag{3.1}$$

In the above the coefficients A and B are defined as

$$\left(\frac{1+ix}{1-ix}\right)^{1/3} = \sum_{n\,\text{even}} A_n x^n + i\sum_{n\,\text{odd}} A_n x^n\,, \qquad \left(\frac{1+ix}{1-ix}\right)^{2/3} = \sum_{n\,\text{even}} B_n x^n + i\sum_{n\,\text{odd}} B_n x^n\,. \tag{3.2}$$

The ghost sector of OSFT in the Siegel gauge is invariant under an SU$(1,1)$ symmetry having a Z_4 subgroup [14, 15]. This is, in fact, reflected in the symmetry $\widetilde{N}^{rs}_{mn} = \widetilde{N}^{sr}_{nm}$, which implies that $\widetilde{N}^{11}_{mn}$ is symmetric, as manifest in (3.1). For a string field built as a bilinear in ghosts and antighosts acting on $c_1|0\rangle$, the condition of SU$(1,1)$ invariance reduces to a Z_4 invariance [14]: $b_{-n}\to -nc_{-n}$, $c_{-n}\to \frac{1}{n}b_{-n}$. As noted in the introduction below (1.1), the proposed universal part of the string field is built as linear superpositions of terms of the form $(nc_{-n}b_{-m} + mc_{-m}b_{-n})$ acting on $c_1|0\rangle$, and these are readily seen to be singlets. Since SU(1,1) is a symmetry even after level truncation it need not be tested further and it will suffice for us to test the universality of the ratios $r_{n,m}$ for $n \geq m$.

To facilitate comparison with numerical results, we now list the predicted values of $r_{n,m}$ for various values of m, n up to $m + n \leq 10$. Using (2.14) and the above expressions for the Neumann coefficients, we find

$$\begin{aligned}
\bar{r}_{1,1} &= \frac{11}{27} \simeq 0.407407\,, & \bar{r}_{3,1} &= -\frac{80}{729} \simeq -0.109739\,,\\
\bar{r}_{5,1} &= \frac{1136}{19683} \simeq 0.0577148\,, & \bar{r}_{3,3} &= \frac{2099}{98415} \simeq 0.021328\,,\\
\bar{r}_{7,1} &= -\frac{6640}{177147} \simeq -0.037483\,, & \bar{r}_{5,3} &= -\frac{17840}{1240029} \simeq -0.0143868\,,\\
\bar{r}_{9,1} &= \frac{388336}{14348907} \simeq 0.0270638\,, & \bar{r}_{7,3} &= \frac{455728}{43046721} \simeq 0.0105868\,,\\
\bar{r}_{5,5} &= \frac{94979}{14348907} \simeq 0.00661925\,. & &
\end{aligned} \tag{3.3}$$

In order to show that $r_{n,m}$ for even m, n are not universal in general, we also give the value of $\bar{r}_{2,2}$ computed according to eq. (2.14). It is

$$\bar{r}_{2,2} = -\frac{19}{729} \simeq -0.0260631\,. \tag{3.4}$$

3.1 Tachyon vacuum solution

For the tachyon vacuum solution we present the results for the ratio $r_{m,n}$ at level $(L, 3L)$ approximation for various values of L, and also extrapolate the results to $L = \infty$ using a linear fit of the form: $f(L) = a + b/L$ with constants a, b. The results are shown in tables 1, 2, and are clearly in good agreement with the predicted values (3.3). Indeed in table 1 the projections differ by about 1% or less from the predictions. This is also the case for the first two columns in table 2. Even for the cases where there is just one data point the values are surprisingly close to the predictions.

In order to demonstrate that the coefficient $r_{2,2}$ does not follow the prediction (3.4), we now quote the level (10,30) result for this ratio. It is:

$$r_{2,2} = -0.0640389\,. \tag{3.5}$$

This is quite far from the prediction (3.4).

The above results show that the vacuum solution gives coefficients $r_{m,n}$ for odd m, n that are consistent with the predictions. We now investigate further the universality of the predictions by examining other solutions of OSFT.

3.2 Tachyon lump solution

In this subsection we present the results for $r_{m,n}$ for the codimension one tachyon lump solution. Unfortunately due to lack of results beyond level (4,8), we can carry out the analysis of this and the next subsection only for fields up to level 4, i.e. for the coefficients $r_{1,1}$ and $r_{3,1}$.

L	$r_{1,1}$	$r_{3,1}$	$r_{5,1}$	$r_{3,3}$
4	0.375042	−0.102499		
6	0.386571	−0.104743	0.0547758	0.0208371
8	0.393062	−0.105966	0.05544	0.0209129
10	0.397214	−0.106707	0.0558499	0.0209927
∞	0.411545	−0.109469	0.0574564	0.0212121
conj	0.407407	−0.109739	0.0577148	0.021328

Table 1: The numerical results for the coefficients $r_{n,m}$ for the tachyon vacuum solution at level $(L, 3L)$ approximation, their extrapolation to $L = \infty$ via a fit $a + b/L$, and the conjectured values.

L	$r_{7,1}$	$r_{5,3}$	$r_{9,1}$	$r_{7,3}$	$r_{5,5}$
8	−0.0358037	−0.014195			
10	−0.0361037	−0.0142012	0.0259407	0.0104886	0.00658638
∞	−0.0373037	−0.014226			
conj	−0.037483	−0.0143868	0.0270638	0.0105868	0.00661925

Table 2: The numerical results for the coefficients $r_{n,m}$ for the tachyon vacuum solution at level $(L, 3L)$ approximation, their extrapolation to $L = \infty$ via a fit $a + b/L$, and the conjectured ratios.

As in [3], we compactify the direction transverse to the lump on a circle of radius R so that the momentum in that direction is quantized in units of $1/R$, and define the level of a field to be the total L_0 eigenvalue of the corresponding state plus one. Since we are interested in studying the solution for different values of R, in order to carry out the truncation to a given level, we need to work with different sets of fields and interaction terms for different values of R. We choose, however, to work with a fixed approximation to the Lagrangian that includes:

1. All fields up to level 4 and interactions up to level 8 in the zero momentum sector.
2. Modes of the tachyon carrying momentum $\pm n/R$ for $n \leq 3$ and all interaction terms involving them.
3. All fields carrying momentum $\pm n/R$ and with level $2 + n^2/R^2$ with $n = 1$ or 2, and the interactions among these and the other fields listed above provided the total level $a + b/R^2$ of all the fields satisfies $a + b/3 < 8$.

This choice of interactions ensures that for $R^2 < 3$, the approximation includes all the fields up to level 4 and interactions up to level 8. We present the results for $r_{1,1}$ and $r_{3,1}$ computed with this action for various values of R in the range $(1, \sqrt{3})$ in table 3. The results are again in good agreement with the predictions (3.3).[1] For comparison we have also included the results for $r_{2,2}$. The table clearly shows the lack of universality of this coefficient.

[1]Although for larger values of R the ratio $r_{1,1}$ deviates from the expected value 0.407407 by about 10%, we note from table 1 that at level (4,12) the vacuum solution ratio $r_{1,1}$ also deviates from the conjectured value by about 10%.

R^2	1.05	1.1	1.5	2	2.5	2.9
$r_{1,1}$	0.400841	0.39765	0.38592	0.379426	0.376123	0.374565
$r_{3,1}$	−0.108368	−0.108068	−0.107668	−0.10666	−0.105063	−0.103675
$r_{2,2}$	−0.0312319	−0.0336458	−0.0422756	−0.0471878	−0.0497961	−0.0510013

Table 3: The numerical results for the coefficients $r_{n,m}$ for the tachyon lump solution for different values of the radius R of the compactified direction transverse to the lump.

λ	.05	.1	.2	.3	.32
$r_{1,1}$	0.407124	0.406234	0.401951	0.39045	0.38568
$r_{3,1}$	−0.109656	−0.109408	−0.108443	−0.107	−0.106524
$r_{2,2}$	−0.0262904	−0.0270022	−0.0303961	−0.0394409	−0.0432926

Table 4: The numerical results for the coefficients $r_{n,m}$ for the solution representing marginal deformation by a Wilson line for different values of the deformation parameter λ.

3.3 Marginal deformations

The set-up here is that of ref. [5]. We choose one particular coordinate tangential to the D-brane which we call X, and take the marginal operator V to be ∂X. Thus the corresponding state $|V\rangle_m$ is given by $\alpha^X_{-1}|0\rangle_m$, where α^X_n denote the oscillators associated with the field X. We take the Siegel gauge string field $|\Psi\rangle$ to be of the form:

$$|\Psi\rangle = \lambda\, \alpha^X_{-1}|0\rangle_m \otimes c_1|0\rangle_g + |\chi\rangle\,, \tag{3.6}$$

where $|\chi\rangle$ satisfies

$$\langle\chi|c_0(\alpha^X_{-1}|0\rangle_m \otimes c_1|0\rangle_g) = 0\,. \tag{3.7}$$

We then determine $|\chi\rangle$ as a function of λ by solving the components of the OSFT equations of motion along every state *except along* $\alpha^X_{-1}|0\rangle_m \otimes c_1|0\rangle_g$. The general philosophy behind this procedure is that while we expect the effective potential for λ to vanish in the full string field theory, at any finite level approximation there is a potential for λ. Thus the component of the string field equations along $\alpha^X_{-1}|0\rangle_m \otimes c_1|0\rangle_g$ will not be satisfied at finite level approximation for arbitrary λ. However, since this is an artifact of level truncation, we do not insist on satisfying the equations of motion along this particular direction in the field space.

We work at level (4,8) for different values of λ and compute the ratios $r_{1,1}$ and $r_{3,1}$ for the solution. The results are reported in table 4. We again find good agreement with the predictions (3.3). In this context we note that the agreement of these results with (3.3) for small λ is automatic since that is how we arrived at these predictions in the first place. But the agreement for finite λ provides a non-trivial check on the universality hypothesis.[2] We have also displayed in the table the ratios $r_{2,2}$ computed in this approximation for different values of λ. The table clearly shows the lack of universality of these coefficients.

[2]Note that $\lambda \simeq .33$ is the limiting value beyond which this procedure of obtaining the solution of the string field equations of motion breaks down[5].

4. Possible origin of the universality

The universality discussed in the previous section is quite surprising because given the non-linear nature of the string field equations of motion, the universal terms receive contribution from the general non-universal terms. This seems to suggest that the solution of the string field equation satisfies a set of linear equations which determine the universal part of the solution. One set of linear constraints was already used to test the consistency of the universal sector. The universal sector is SU(1,1) invariant and thus annihilated by the SU(1,1) generators.

At present we do not know of any way to generate a set of linear equations which completely determine the universal part of the solution, but we shall now present two sets of linear equations of this type. We begin with the string field equation:

$$Q_B|\Psi\rangle + |\Psi * \Psi\rangle = 0\,, \tag{4.1}$$

and apply $c(\pm i)$ on both sides. Since c is an operator of negative dimension, the action of $c(\pm i)$ on the $*$-product of two states vanishes.[3] This gives

$$c(\pm i) Q_B|\Psi\rangle = 0\,. \tag{4.2}$$

(Related identities were discussed in ref.[17].) We note in passing that since $c\partial c$ is also of negative dimension the equation $c\partial c(\pm i)Q_B|\Psi\rangle = 0$ must also hold.

Applying b_0 on both sides of (4.2), and using the results:

$$b_0|\Psi\rangle = 0\,, \qquad \{b_0, c(\pm i)\} = \mp i\,, \qquad \{b_0, Q_B\} = L_0\,, \tag{4.3}$$

we get

$$(\mp i Q_B - c(\pm i)L_0)|\Psi\rangle = 0\,. \tag{4.4}$$

Taking the sum and differences of these equations, and the expansion

$$c(z) = \sum_n c_n z^{-n-1}\,, \tag{4.5}$$

we get the two linear conditions on $|\Psi\rangle$:

$$\sum_{m=0}^{\infty} (-1)^m (c_{2m+1} - c_{-2m-1}) L_0 |\Psi\rangle = 0\,, \tag{4.6}$$

and

$$\left(Q_B - c_0 L_0 - \sum_{m=1}^{\infty} (-1)^m (c_{-2m} + c_{2m}) L_0 \right) |\Psi\rangle = 0\,. \tag{4.7}$$

[3] This is true for Fock space states and arises because the conformal map needed for a midpoint insertion produces a factor of zero for the case of a negative dimension operator (see, for example the related discussion in section 2.1 of [7]). We shall proceed by assuming that this holds for all *allowed* string field configurations. It is tempting to conjecture that validity of this condition can be taken as the criterion to determine which string field configurations are allowed.

Note in particular that the contribution to the left hand side of eq. (4.6) along states of the form $c_{-p}c_1|0\rangle$ for odd p involves only the components of $|\Psi\rangle$ along states of the form $c_{-n}b_{-m}c_1|0\rangle$ for odd m,n. Since these coefficients are conjectured to be given by eq. (2.14), one could ask if they satisfy eq. (4.6). Using eqs. (2.14), (4.6) we get,

$$(2n+1)\sum_{m=0}^{\infty}(-1)^m \widetilde{N}^{11}_{(2m+1),(2n+1)} = (-1)^n \,. \tag{4.8}$$

To establish this equality, we now recall that the relevant matrices in the ghost vertex satisfy [13, 18, 11]

$$\widetilde{V}^{11}_{nm} = \widetilde{N}^{11}_{mn} n\,, \qquad \widetilde{M} = C\widetilde{V}^{11} = -E\left(\frac{M}{1+2M}\right)E^{-1}\,, \qquad E_{mn} = \sqrt{m}\delta_{mn}\,, \tag{4.9}$$

where M is the matrix associated with matter Neumann coefficients, defined in [18]. Equation (4.8) then becomes

$$\sum_{m=0}^{\infty}\widetilde{V}^{11}_{(2n+1),(2m+1)}(-1)^m = -(-1)^n\,, \tag{4.10}$$

which is an eigenvalue equation for a vector v:

$$\sum_{m=0}^{\infty}\widetilde{V}^{11}_{nm}v_m = -v_n\,, \qquad v_{2m+1} = (-1)^m\,, \qquad v_{2m} = 0\,. \tag{4.11}$$

Indeed this equation is satisfied as we now explain. Using (4.9), we see that the C-odd eigenvector v^- of M with eigenvalue $-1/3$ satisfies $\widetilde{V}^{11}(Ev^-) = -(Ev^-)$. One then readily confirms that $v = Ev^-$ using the known components of v^- [20, 18].

A direct proof of (4.8) can be given as follows. We have the equation:

$$b_0\,(c(i)+c(-i))((c_1|0\rangle) * (c_1|0\rangle)) = 0\,. \tag{4.12}$$

From this we can derive eq. (4.8) by expressing the $*$-product on the left hand side of eq. (4.12) in terms of the ghost Neumann coefficients, and then setting to zero the coefficient of $c_{-p}c_1|0\rangle$ for odd p. Alternatively we can argue that since the coefficients $r_{n,m}$ given in (2.14) explicitly appear in the order λ^2 solution generated by a marginal deformation, and since this particular solution also satisfies the identities (4.6), (4.7), eq. (2.14) must be consistent with the identities (4.6), (4.7).

One could also ask if it is possible to check the other identities following from eqs. (4.6) and (4.7) using the level truncated solution. Unfortunately the convergence of these relations is not sufficiently fast to allow us to draw any definite conclusion. In particular even in the simplest case of $n=0$, the contribution to the left hand side of eq. (4.8) from terms up to level 14 (e.g. with $m \le 6$) only gives about 68% of the expected value of one. Going up to level 200 produces about 87% of the expected answer.

5. Approximate factorization of $L_0|\Psi_0\rangle$

In vacuum string field theory, the solutions of classical equations of motion factorize into a product of a state in the matter state space and another state in the ghost state space. Since in OSFT the kinetic operator Q_B receives contribution from both the ghost and the matter sector, we do not expect the solutions to have such simple factorization property. While the vacuum solution $|\Psi_0\rangle$ is far from factorized, $L_0|\Psi_0\rangle$ seems to possess some approximate factorization property which we shall demonstrate now.

We begin by giving $L_0|\Psi_0\rangle$, calculated at level (10,30), up to level 6:

$$\begin{aligned}
L_0|\Psi_0\rangle \; = \; -0.54626\Big[&1 - 0.10478L^m_{-2} + 0.39721b_{-1}c_{-1} + 0.02798L^m_{-4} + \\
&+ 0.00354L^m_{-2}L^m_{-2} - \underline{0.04322}L^m_{-2}b_{-1}c_{-1} - 0.32011b_{-1}c_{-3} - \\
&- 0.19211b_{-2}c_{-2} - 0.10670b_{-3}c_{-1} - 0.01327L^m_{-6} - \\
&- 0.00282L^m_{-4}L^m_{-2} - 0.00006L^m_{-3}L^m_{-3} + 0.00007L^m_{-2}L^m_{-2}L^m_{-2} + \\
&+ \underline{0.01110}L^m_{-4}b_{-1}c_{-1} + \underline{0.00173}L^m_{-2}L^m_{-2}b_{-1}c_{-1} + \underline{0.00068}L^m_{-3}b_{-1}c_{-2} + \\
&+ \underline{0.00034}L^m_{-3}b_{-2}c_{-1} + \underline{0.03217}L^m_{-2}b_{-1}c_{-3} + \underline{0.02708}L^m_{-2}b_{-2}c_{-2} + \\
&+ \underline{0.01072}L^m_{-2}b_{-3}c_{-1} + 0.27924b_{-1}c_{-5} + 0.17495b_{-2}c_{-4} + \\
&+ 0.10496b_{-3}c_{-3} - 0.07393b_{-2}b_{-1}c_{-1}c_{-2} + 0.08747b_{-4}c_{-2} + \\
&+ 0.05584b_{-5}c_{-1}\Big]c_1|0\rangle\,.
\end{aligned} \tag{5.1}$$

In the above, we have underlined the terms that involve both matter and ghost operators. If $L_0|\Psi_0\rangle$ had a factorized form, then we could determine the state by simply looking at the pure matter and pure ghost excitations in $L_0|\Psi_0\rangle$ and then taking their direct product. Calling $(L_0|\Psi_0\rangle)_0$ the state assembled in this way from the above equation, we have:

$$\begin{aligned}
(L_0|\Psi_0\rangle)_0 = \; &-0.54626\Big(1 + 0.39721b_{-1}c_{-1} - 0.32011b_{-1}c_{-3} - \\
&\qquad - 0.19211b_{-2}c_{-2} - 0.10670b_{-3}c_{-1} + \cdots\Big) \times \\
&\times(1 - 0.10478L^m_{-2} + 0.02798L^m_{-4} + 0.00354L^m_{-2}L^m_{-2} + \cdots)c_1|0\rangle \\
= \; &-0.54626\Big[1 - 0.10478L^m_{-2} + 0.39721b_{-1}c_{-1} + 0.02798L^m_{-4} + \\
&+ 0.00354L^m_{-2}L^m_{-2} - \underline{0.04162}L^m_{-2}b_{-1}c_{-1} - 0.32011b_{-1}c_{-3} - \\
&- 0.19211b_{-2}c_{-2} - 0.10670b_{-3}c_{-1} - 0.01327L^m_{-6} - 0.00282L^m_{-4}L^m_{-2} - \\
&- 0.00006L^m_{-3}L^m_{-3} + 0.00007L^m_{-2}L^m_{-2}L^m_{-2} + \\
&+ \underline{0.01111}L^m_{-4}b_{-1}c_{-1} + \underline{0.00141}L^m_{-2}L^m_{-2}b_{-1}c_{-1} + \underline{0.0}L^m_{-3}b_{-1}c_{-2} + \\
&+ \underline{0.0}L^m_{-3}b_{-2}c_{-1} + \underline{0.03354}L^m_{-2}b_{-1}c_{-3} + \underline{0.02013}L^m_{-2}b_{-2}c_{-2} + \\
&+ \underline{0.01118}L^m_{-2}b_{-3}c_{-1} + 0.27924b_{-1}c_{-5} + 0.17495b_{-2}c_{-4} + \\
&+ 0.10496b_{-3}c_{-3} - 0.07393b_{-2}b_{-1}c_{-1}c_{-2} + 0.08747b_{-4}c_{-2} + \\
&+ 0.05584b_{-5}c_{-1}\Big]c_1|0\rangle\,.
\end{aligned} \tag{5.2}$$

We now compare the expressions on the right hand sides of eq. (5.1) and (5.2). By construction, the pure matter and pure ghost terms are identical. The underlined terms,

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however, test the factorization property since they arise from products. Some terms are remarkably accurate (like the $L^m_{-4}b_{-1}c_{-1}$, with error of one part in a thousand), several are within about 5%, and a couple exceed 20% error. Experiments with lower level results indicate that the coefficients do not always approach each other as we increase the level. If factorization held exactly, terms as $L_{-3}b_{-2}c_{-1}$ would have to vanish since because of twist property the string field cannot have expectation values for the separate matter and ghost parts. Indeed, the coefficients of such terms in $L_0|\Psi_0\rangle$ are small, but they do not seem to go to zero as the level is increased. Overall, despite striking patterns, we conclude that $L_0|\Psi_0\rangle$ is at best approximately factorized.

Since the star product of factorized fields is factorized, an approximate factorization for $L_0|\Psi_0\rangle$ would arise if the dominant contribution to $b_0|\Psi_0 * \Psi_0\rangle$ (and hence, $L_0|\Psi_0\rangle$) came from a part of $|\Psi_0\rangle$ that is factorized. To see if this is the origin of factorization, we have taken the level 10 expression for $|\Psi_0\rangle$, identified its pure matter and pure ghost excitations, and defined a new configuration $|\Phi_0\rangle$ by taking the direct product of these two factors. The result for $|\Phi_0 * \Phi_0\rangle$ is given below:

$$\begin{aligned}
-b_0|\Phi_0 * \Phi_0\rangle = 0.45634\Big[&1 - 0.06884L^m_{-2} + -0.39105b_{-1}c_{-1} + 0.02722L^m_{-4} - \\
&- 0.00170L^m_{-2}L^m_{-2}\underline{0.02707}L^m_{-2}b_{-1} - c_{-1}0.32889b_{-1}c_{-3} - \\
&- 0.15360b_{-2}c_{-2} - 10963b_{-3}c_{-1} - 0.01305L^m_{-6} - \\
&- 0.00161L^m_{-4}L^m_{-2} - 0.00014L^m_{-3}L^m_{-3} + 0.00042L^m_{-2}L^m_{-2}L^m_{-2} \\
&+ \underline{0.01067}L^m_{-4}b_{-1}c_{-1} - \underline{0.00064}L^m_{-2}L^m_{-2}b_{-1}c_{-1} + \underline{0.0}L^m_{-3}b_{-1}c_{-2} + \\
&+ \underline{0.0}L^m_{-3}b_{-2}c_{-1} + \underline{0.02277}L^m_{-2}b_{-1}c_{-3} + \underline{0.01059}L^m_{-2}b_{-2}c_{-2} + \\
&+ \underline{0.00759}L^m_{-2}b_{-3}c_{-1} + 0.28947b_{-1}c_{-5} + 0.14175b_{-2}c_{-4} + \\
&+ 0.10522b_{-3}c_{-3} - 0.05942b_{-2}b_{-1}c_{-1}c_{-2} + 0.07087b_{-4}c_{-2} \\
&+ 0.05789b_{-5}c_{-1}\Big]c_1|0\rangle\,. \qquad (5.3)
\end{aligned}$$

Comparing eqs. (5.1)–(5.3) we see that (5.2) is closer to (5.1) than (5.3) is to (5.1). Thus the explanation for the approximate factorization *does not* completely lie in the fact that the factorized part of $|\Psi_0\rangle$ gives the dominant contribution to $|\Psi_0 * \Psi_0\rangle$. Somehow, products of factorized terms times mixed terms, and mixed terms times mixed terms do give substantial factorized contributions.

6. A surprising coincidence in the matter sector

In previous sections we discussed a possible universality of the ghost part of the OSFT field equations. This analysis shows that at least some part of the solution involving ghost excitations may not depend on which particular background the solution describes. In this section we shall discuss a different kind of universality in the matter sector where we shall show that two different field equations which differ from each other in their ghost structure have solutions whose pure matter parts appear to be quite close to each other.

The OSFT field equation in the Siegel gauge takes the form:

$$L_0|\Psi\rangle = -b_0|\Psi * \Psi\rangle\,, \qquad (6.1)$$

where $|\Psi\rangle$ is a ghost number 1 field equation. The solution of this equation representing the tachyon vacuum solution is a linear combination of states, obtained by matter Virasoro generators $L_{-k}^{(m)}$ and ghost oscillators b_{-m}, c_{-n} acting on $c_1|0\rangle$[16]. Let us focus on the part of the solution involving pure matter excitations. If we denote by $\mathcal{O}_s(L^{(m)})$ some combination of matter Virasoro generators, then the pure matter part of the solution takes the form:

$$\alpha_0 \left(c_1|0\rangle + \sum_s \beta_s \mathcal{O}_s(L^{(m)}) c_1|0\rangle \right), \tag{6.2}$$

where α_0, β_s are constants. We adopt the convention that $\beta_{-m_1,\ldots,-m_r}$ will denote the coefficient of $L_{-m_1}^{(m)} \cdots L_{-m_r}^{(m)} c_1|0\rangle$, with $m_1 \geq m_2 \geq \cdots \geq m_r$.

Since L_0 acting on the ghost vacuum $c_1|0\rangle$ has eigenvalue -1, a closely related equation for a ghost number zero field $|\Phi\rangle$ is:

$$(L_0 - 1)|\Phi\rangle = -|\Phi * \Phi\rangle\,. \tag{6.3}$$

We can look for a solution of this equation of the form:

$$|\Phi_0\rangle = |0\rangle + \sum_s \gamma_s \mathcal{O}_s(L)|0\rangle\,, \tag{6.4}$$

where γ_s are constants and L denotes the total Virasoro generators of the matter-ghost system. We again use the convention that $\gamma_{-m_1,\ldots,-m_r}$ denotes the coefficient of $L_{-m_1} \cdots L_{-m_r}|0\rangle$. Note that since the Virasoro generators L_m have vanishing central charge, the coefficient of $|0\rangle$ in eq. (6.4) is unity. From eq. (6.4) it follows that part of $|\Phi_0\rangle$ involving pure matter excitations is given by:

$$|0\rangle + \sum_s \gamma_s \mathcal{O}_s(L^{(m)})|0\rangle\,. \tag{6.5}$$

Examining the numerical solutions of eqs. (6.1) and (6.3) using level truncation, we find the following correspondence between the coefficients β_s and γ_s:

$$\beta_s \simeq \gamma_s\,. \tag{6.6}$$

The results are presented in appendix A. For most of the coefficients the correspondence seems to be valid to within a few percent at level 10. There are some exceptions, e.g. the coefficients of $(L_{-2})^n$, with $n \geq 3$. However we note from the tables in appendix A that at a given level, the coefficients of such terms are order of magnitude smaller than the leading terms at this level, e.g. the coefficient of L_{-2n}. Indeed we see from these tables that the error in the coefficients of $(L_{-2})^n$ is of the same order of magnitude as the error in the coefficients of L_{-2n}.

7. Concluding remarks

One lesson we have learned in the present investigation is that "quasi-patterns" seem to exist – these are remarkable coincidences having a theoretical ring to them that appear to hold closely in the level expansion but do not truly hold exactly. A pattern may be only a quasi-pattern when successive level calculations do not appear to improve systematically the accuracy. Perhaps the first example of such quasi-pattern was the zero norm property of ref. [14], the accuracy of which stops improving at level eight.

We have provided evidence that certain ratios of expectation values in the purely ghost sector of all (known) solutions of OSFT take fixed values given by simple expressions in terms of Neumann coefficients. While we have no full understanding of the theoretical meaning of this observation, we showed that linear constraints on the string field give partial consistency checks and thus some analytic evidence for the proposal. The numerical evidence also seems to improve with level. All in all we feel that there is some strong but not overwhelming evidence for this to be an exact pattern.

For the case of the correspondence between ghost number zero and ghost number one field equations, the near equality of expectation values is quite striking, but the pattern is somewhat irregular and lacking an analytic understanding for how this correspondence could arise, it seems premature to propose a strict equality. In particular, for some coefficients (e.g. of L_{-4}) where the results are fairly stable in level expansion, the correspondence still has an error of about 2%. Higher level data could help us reach a conclusion, but this could be a quasi-pattern.

Finally, the factorization of $L_0\Psi_0$ for the tachyon vacuum solution would seem to be a quasi-pattern. Given the theoretical significance of factorized states (such as surface states) the factorization pattern would be remarkable if exact, but the evidence again is that some small errors do not seem to get smaller as we proceed at higher levels.

It is hoped that the observations and patterns noted in this work will help guide in the search for exact analytic solutions of OSFT.

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A. Comparison of β_s and γ_s

In this appendix we shall give the results for the coefficients β_s and γ_s at various levels of approximation. The $L = \infty$ answer given in each table is obtained using an extrapolation in L with a function $a + b/L$.

L	β_{-2}	γ_{-2}		β_{-4}	γ_{-4}		$\beta_{-2,-2}$	$\gamma_{-2,-2}$
4	0.103799	0.106693		-0.009338	-0.00941869		-0.00107467	-0.00119004
6	0.104289	0.106822		-0.00932407	-0.00945266		-0.00113529	-0.00121053
8	0.104586	0.106864		-0.0093259	-0.00946405		-0.00116427	-0.00121627
10	0.104787	0.106882		-0.00932777	-0.00946906		-0.00118174	-0.00121856
∞	0.10541	0.107021		-0.00931707	-0.009506		-0.00125357	-0.00124007

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L	β_{-6}	γ_{-6}		$\beta_{-4,-2}$	$\gamma_{-4,-2}$
6	0.00266208	0.00267922		0.00055773	0.000588318
8	0.00265609	0.00268466		0.000562876	0.000591376
10	0.00265563	0.00268703		0.000565538	0.000592452
∞	0.00264468	0.00269687		0.000577424	0.000598963

L	$\beta_{-3,-3}$	$\gamma_{-3,-3}$		$\beta_{-2,-2,-2}$	$\gamma_{-2,-2,-2}$
6	1.1188×10^{-5}	1.14982×10^{-5}		-1.80854×10^{-5}	-1.81025×10^{-5}
8	1.27096×10^{-5}	1.30298×10^{-5}		-1.56105×10^{-5}	-1.74459×10^{-5}
10	1.3302×10^{-5}	1.35892×10^{-5}		-1.42765×10^{-5}	-1.72127×10^{-5}
∞	1.66038×10^{-5}	1.68725×10^{-5}		-0.850154×10^{-5}	-1.58124×10^{-5}

L	β_{-8}	γ_{-8}		$\beta_{-6,-2}$	$\gamma_{-6,-2}$
8	-0.00104572	-0.00104938		-0.000190741	-0.000199537
10	-0.00104339	-0.00105092		-0.000191915	-0.000200248
∞	-0.00103407	-0.00105708		-0.000196611	-0.000203092

L	$\beta_{-5,-3}$	$\gamma_{-5,-3}$		$\beta_{-4,-4}$	$\gamma_{-4,-4}$
8	-3.30137×10^{-6}	-3.21237×10^{-6}		-5.50356×10^{-5}	-5.67779×10^{-5}
10	-3.76634×10^{-6}	-3.73539×10^{-6}		-5.51104×10^{-5}	-5.69378×10^{-5}
∞	-5.62622×10^{-6}	-5.82747×10^{-6}		-5.54096×10^{-5}	-5.75774×10^{-5}

L	$\beta_{-4,-2,-2}$	$\gamma_{-4,-2,-2}$		$\beta_{-3,-3,-2}$	$\gamma_{-3,-3,-2}$
8	-1.09361×10^{-5}	-1.25777×10^{-5}		-1.35659×10^{-6}	-1.49837×10^{-6}
10	-1.14282×10^{-5}	-1.27416×10^{-5}		-1.50681×10^{-6}	-1.62021×10^{-6}
∞	-1.33966×10^{-5}	-1.33972×10^{-5}		-2.10769×10^{-6}	-2.10757×10^{-6}

L	$\beta_{-2,-2,-2,-2}$	$\gamma_{-2,-2,-2,-2}$
8	1.81597×10^{-6}	2.08577×10^{-6}
10	1.73594×10^{-6}	2.06738×10^{-6}
∞	1.41582×10^{-6}	1.99382×10^{-6}

L	β_{-10}	γ_{-10}		$\beta_{-8,-2}$	$\gamma_{-8,-2}$
10	0.000533168	0.00053466		8.26156×10^{-5}	8.57127×10^{-5}

L	$\beta_{-7,-3}$	$\gamma_{-7,-3}$		$\beta_{-6,-4}$	$\gamma_{-6,-4}$
10	7.80062×10^{-7}	7.17496×10^{-7}		4.07104×10^{-5}	4.18855×10^{-5}

L	$\beta_{-5,-5}$	$\gamma_{-5,-5}$		$\beta_{-6,-2,-2}$	$\gamma_{-6,-2,-2}$
10	3.14321×10^{-7}	2.84212×10^{-7}		4.23563×10^{-6}	4.70925×10^{-6}

L	$\beta_{-5,-3,-2}$	$\gamma_{-5,-3,-2}$		$\beta_{-4,-4,-2}$	$\gamma_{-4,-4,-2}$
10	4.48058×10^{-7}	4.83439×10^{-7}		4.097×10^{-6}	4.44877×10^{-6}

L	$\beta_{-4,-3,-3}$	$\gamma_{-4,-3,-3}$		$\beta_{-4,-2,-2,-2}$	$\gamma_{-4,-2,-2,-2}$
10	1.89101×10^{-7}	1.96492×10^{-7}		-4.13579×10^{-7}	-4.29091×10^{-7}

L	$\beta_{-3,-3,-2,-2}$	$\gamma_{-3,-3,-2,-2}$		$\beta_{-2,-2,-2,-2,-2}$	$\gamma_{-2,-2,-2,-2,-2}$
10	8.47716×10^{-8}	9.94688×10^{-8}		-6.62409×10^{-8}	-8.43162×10^{-8}

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